

Short Course in Mathematics and Analytic Geometry

Week 8 Exercises

1 Complex Numbers

Simplify each of the following problems:

- | | | |
|----------------------|---|-------------------------------------|
| 1. $\frac{1}{1+i}$ | 2. $\frac{1}{i-1}$ | 3. i^4 |
| 4. $i^2 + 2i + 1$ | 5. $(i + \sqrt{3})^2$ | 6. $\left(\frac{1+i}{1-i}\right)^2$ |
| 7. $\frac{3+1}{2+1}$ | 8. $5(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5})$ | 9. $\frac{5-2i}{5+2i}$ |

Use the relationship $z = re^{\theta i} = r(\cos \theta + i \sin \theta)$ to expand the following:

- | | | |
|-------------------------|-----------------------|---------------------|
| 10. z^t | 11. $\frac{1}{z^2}$ | 12. $\frac{1}{z+1}$ |
| 13. z^5 | 14. $z^{\frac{1}{2}}$ | 15. $(z+1)^2$ |
| 16. $\frac{1}{(z-1)^3}$ | 17. $\frac{z+1}{z}$ | |

Given a quaternion $q = s + \bar{v}$, where s is scalar and \bar{v} is a quaternion vector, use the relationship $e^q = e^s(\cos(|\bar{v}|) + \frac{\bar{v}}{|\bar{v}|} \sin(|\bar{v}|))$ to expand the following:

- | | | |
|-------------------------|-----------------------|---------------------|
| 18. q^t | 19. $\frac{1}{q^2}$ | 20. $\frac{1}{q+1}$ |
| 21. q^5 | 22. $q^{\frac{1}{2}}$ | 23. $(q+1)^2$ |
| 24. $\frac{1}{(q-1)^3}$ | 25. $\frac{q+1}{q}$ | |