

# Short Course in Mathematics and Analytic Geometry

## Week 8 Exercises

### 1 Complex Numbers

Simplify each of the following problems (Solutions):

$$1. \frac{1}{1+i} = \frac{1-i}{(1-i)(1+i)} = \frac{1-i}{2} = \frac{1}{2} - \frac{i}{2}$$

$$2. \frac{1}{i-1} = \frac{-1-i}{(-1-i)(i-1)} = \frac{-1-i}{2} = -\frac{1}{2} - \frac{i}{2}$$

$$3. i^4 = (\sqrt{-1})^4 = 1$$

$$4. i^2 + 2i + 1 = -1 + 2i + 1 = 2i$$

$$5. (i + \sqrt{3})^2 = i^2 + 2\sqrt{3}i + 3 = -1 + 2\sqrt{3}i + 3 = 2 + 2\sqrt{3}i$$

$$6. \left(\frac{1+i}{1-i}\right)^2 = \left(\frac{(1+i)^2}{2}\right)^2 = \frac{(1+i)^4}{4} = \frac{1}{4}(1 + 4i + 6i^2 + 4i^3 + i^4) = \frac{1}{4}(1 + 4i - 6 - 4i + 1) = -1$$

$$7. \frac{3+1}{2+1} = \frac{4}{3}$$

$$8. 5(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}) = 5e^{\frac{2\pi}{5}}$$

$$9. \frac{5-2i}{5+2i} = \frac{(5-2i)^2}{5^2+2^2} = \frac{1}{29}(25 - 20i + 4) = 1 - \frac{20}{29}i$$

Use the relationship  $z = re^{\theta i} = r(\cos \theta + i \sin \theta)$  to expand the following:

$$10. z^t = r^t e^{t\theta i} = r^t (\cos t\theta + i \sin t\theta)$$

$$11. \frac{1}{z^2} = z^{-2} = r^{-2} e^{-2\theta i} = r^{-2} (\cos -2\theta + i \sin -2\theta) = r^{-2} (\cos 2\theta - i \sin 2\theta)$$

$$12. \frac{1}{z+1} = \frac{1}{(r \cos \theta + 1) + ir \sin \theta} = \frac{(r \cos \theta + 1) - ir \sin \theta}{(r \cos \theta + 1)^2 + r^2 \sin^2 \theta} = \frac{(r \cos \theta + 1) - ir \sin \theta}{r^2 \cos^2 \theta + 2r \cos \theta + 1 + r^2 \sin^2 \theta} = \frac{(r \cos \theta + 1) - ir \sin \theta}{r^2 + 2r \cos \theta + 1}$$

$$13. z^5 = r^5 e^{5\theta i} = r^5 (\cos 5\theta + i \sin 5\theta)$$

$$14. z^{\frac{1}{2}} = \sqrt{r} e^{\frac{\theta i}{2}} = \sqrt{r} (\cos \frac{\theta}{2} + i \sin \frac{\theta}{2})$$

$$15. (z+1)^2 = z^2 + 2z + 1 = r^2 e^{2\theta i} + 2r e^{\theta i} + 1 = (r^2 \cos 2\theta + 2r \cos \theta + 1) + i(r^2 \sin 2\theta + 2r \sin \theta)$$

$$16. \frac{1}{(z-1)^3} = \frac{1}{z^3-3z^2+3z-1} = \frac{1}{(r^3 \cos 3\theta - 3r^2 \cos 2\theta + 3r \cos \theta - 1) + i(r^3 \sin 3\theta - 3r^2 \sin 2\theta + 3r \sin \theta)} =$$

$$\frac{(r^3 \cos 3\theta - 3r^2 \cos 2\theta + 3r \cos \theta - 1) - i(r^3 \sin 3\theta - 3r^2 \sin 2\theta + 3r \sin \theta)}{(r^3 \cos 3\theta - 3r^2 \cos 2\theta + 3r \cos \theta - 1)^2 + (r^3 \sin 3\theta - 3r^2 \sin 2\theta + 3r \sin \theta)^2}$$

$$17. \frac{z+1}{z} = 1 + \frac{1}{z} = 1 + z^{-1} = 1 + r^{-1}e^{-\theta i} = 1 + r^{-1}(\cos -\theta + i \sin -\theta) = 1 + r^{-1}e^{-\theta i} = 1 + r^{-1}(\cos \theta - i \sin \theta)$$

\*\*\*Sorry, these was not the intended set of questions for the quaternion problems:\*\*\*

$$18. q^t$$

$$19. \frac{1}{q^2}$$

$$20. \frac{1}{q+1}$$

$$21. q^5$$

$$22. q^{\frac{1}{2}}$$

$$23. (q+1)^2$$

$$24. \frac{1}{(q-1)^3}$$

$$25. \frac{q+1}{q}$$